An Analysis of Turbulent Flow in Concentric Annuli

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Fully developed turbulent flow in a concentric annular geometry is of interest to the chemical engineer because of its many applications in the design of transfer equipment. Consequently, a considerable amount of effort has been devoted to developing expressions that describe the momentum profiles.

Levy (*4*) developed a tedious expression, applicable to both the inner and outer regions, which reduces to the logarithmic velocity distribution law. Roberts (*5*) simplified the expression by incorporating the concept of a variable-mixture length constant. The concept is unacceptable since workers, notably Stevenson (*6*), have indicated that the mixture length remains unchanged even under conditions of severe perturbations in a boundary layer region. Recently, Clump and Kwasnoski (*2*), and Wilson and Medwell (*8*) have developed expressions to predict the velocity distributions in the two regions. Both methods require extensive machine calculations before a satisfactory answer can be obtained. In addition, both expressions deviate from the experimental data of Brighton and Jones (*1*), especially in the region away from the wall.

Accepted forms of universal logarithmic velocity distribution laws, with constants evaluated for a circular pipe flow, have shown good agreement with experimental data in the outer wall region. To date, there exists no simple expression to predict the turbulent velocity profiles in the inner wall region. This paper presents a simple analysis leading to results which agree with experimental results as well if not better than the results of the other more cumbersome expressions.

Using simple force balances, a shear stress distribution can be obtained as a function of the radial coordinates.

$$\beta = \left(\frac{U_{\tau 1}}{U_{\tau 2}}\right)^2 = \frac{r_2}{r_1}\left[\frac{r_m^2 - r_1^2}{r_2^2 - r_m^2}\right] \tag{1}$$

Leung, et al. (*3*) have suggested an expression for determining the radius of maximum velocity, r_m. Roberts (*5*) has found this to be good even for large radius ratios. We have found that this predicts the available experimental data closer than any other tested expression.

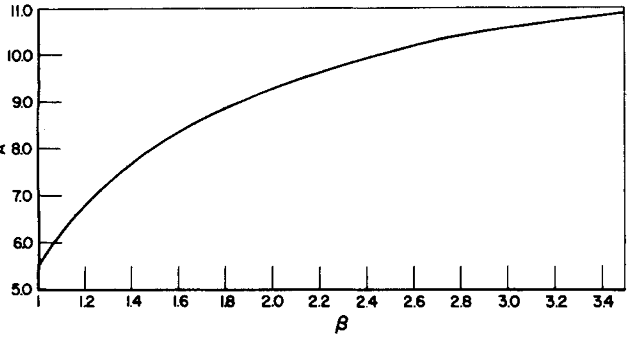

Fig. 1. Variation of A′ with β.

$$r_m = r_1 \left[\frac{1 + \dfrac{r_2^{(1-n)}}{r_1}}{1 + \dfrac{r_1^n}{r_2}}\right] \quad \text{where } n = 0.343 \tag{2}$$

Given r_1 and r_2 one can determine β by using Equations (1) and (2).

In the region near the wall it can be shown that

$$\beta = \frac{dU_2^+}{dy^+} \Big/ \frac{dU_1^+}{dy^+} \tag{3}$$

For the outer wall region we will accept the conventional logarithmic expression:

$$U_2^+ = A + K \ln y^+ \tag{4}$$

Differentiating Equation (4) gives

$$\frac{dU_2^+}{dy^+} = \frac{K}{y^+} \tag{5}$$

Substituting Equation (3) into Equation (5) gives

$$\frac{K}{y^+} = \beta \frac{dU_1^+}{dy^+} \tag{6}$$

Integrating Equation (6) gives

$$U_1^+ = \frac{K}{\beta} \ln y^+ + A' \tag{7}$$

To determine A' we assume that law of the wall is unchanged in both the inner and outer region, and take notice of the fact that all the logarithmic distributions converge at a point. The coordinates of this point, $y^+ = 19.7$ and $U^+ = 13.2$, have been theoretically found by Wasan, et al. (*7*) to be the outer limit of the law of the wall.

Figure 1 shows the variation of A', the constant in Equation (7), as a function of β, which is the square of the ratio of friction velocities.

Figure 2 shows the change in the logarithmic distribution law with an increase of β. When $\beta = 1$, the line predicts the velocity in the outer wall region which is the conventional von Karman profile. Several authors (*2, 5*) have inadvertently shown their dimensionless velocity distributions only as a function of the radius ratios, whereas they must prescribe and indicate one other variable: the inner or outer tube radius.

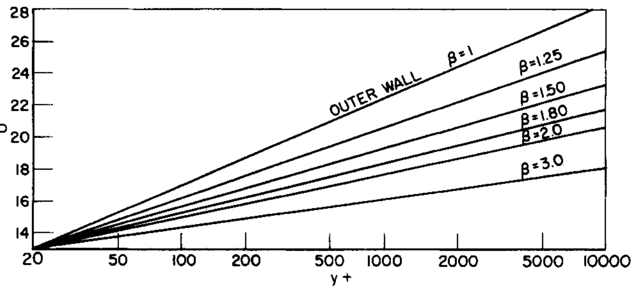

Fig. 2. Effect of increase in β on logarithmic distribution law.

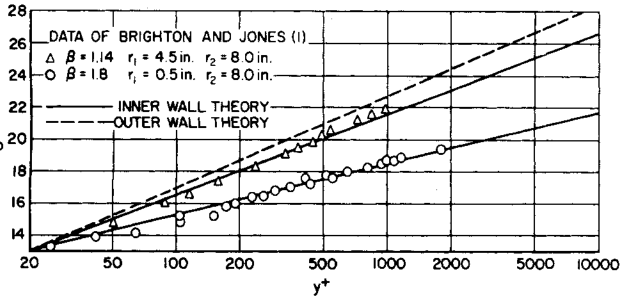

Fig. 3. Comparison of experimental data with proposed theories.

Figure 3 shows a comparison of the experimental data of Brighton and Jones (*1*) with the proposed theory for two values of β. As shown in the figure, there is a very good agreement between the data and the theoretical distribution.

NOTATION

n = parameter defined in Equation (2)
r = radial distance, ft.
r_m = radius of maximum velocity, ft.
U = time-averaged local axial velocity, ft./sec.
U_τ = friction velocity, ft./sec.
U^+ = dimensionless velocity, u/u_r
y = distance from wall, ft.
y^+ = dimensionless distance, $y\ u_r/v$

Subscripts

1 = inner surface of the annulus
2 = outer surface of the annulus
m = portion of maximum velocity

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On the Height of Incipience of Boiling in Tubes of Small Diameter for a Liquid in Laminar Flow

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One of the factors influencing the overall rate of heat transfer to a liquid flowing in a tube is the point (if any) where nucleate boiling begins. This problem has been considered by Davis and Anderson (*1*) for a turbulent flow situation where the bubble nuclei develop within the laminar sublayer where linear temperature profiles can be assumed. However for tubes of small diameter, such as are used in some nuclear and aerospace applications, the liquid is often in laminar flow and there is need for a technique to predict incipience of boiling for this flow regime. Siegel, Sparrow, and Hallman (*2*) have obtained the solution for the temperature distribution in a liquid flowing in a vertical tube (at positions sufficiently removed from the entrance to allow the neglect of entrance effects) of constant radius. Liquid properties were assumed to be independent of temperature and a constant heat flux at the wall was assumed.

$$\frac{T-T_o}{(qr_o/k)} = \frac{4(z/r_o)}{N_{Re}N_{Pr}} + \left(\frac{r}{r_o}\right)^2 - \frac{1}{4}\left(\frac{r}{r_o}\right)^4 - \frac{7}{24} + \sum_{n=1}^{\infty} C_n\, e^{-\frac{\beta_n^2}{N_{Re}N_{Pr}}\left(\frac{z}{r_o}\right)} R_n\left(\frac{r}{r_o}\right) \quad (1)$$

The values of C_n, β_n, and $R_n(r/r_o)$ were obtained numerically and the first seven values of C_n and β_n are given in their paper. Tables of the first four eigenfunctions, $R_n(r/r_o)$, were obtained from Siegel (*3*).

By using the commonly accepted theory that the bubbles originate at small cavities in the system walls, one can write the following equation for the segment of a spherical bubble at the wall at equilibrium.

$$P_g - P_l = \frac{2\sigma}{r_b} \quad (2)$$

If the vapor is assumed to be a perfect gas, the Clausius-Clapeyron equation can be used to replace the vapor pressure in Equation (2) with temperatures, to obtain

$$T_g - T_s = \left(\frac{T_g T_s R}{\lambda}\right) \ln\left(1 + \frac{2\sigma}{r_b P_1}\right) \quad (3)$$

Assuming that the bubble at the start of its growth is a hemisphere and following the reasoning of Bergles (*4*), the following conditions must be met before the bubble will grow.

$$T_g = T \text{ at } r_b = y$$

$$\frac{\partial T_g}{\partial r_b} = \frac{\partial T}{\partial y} \text{ at } r_b = y$$

$$y = r_o - r$$

Applying these conditions to Equations (1) and (3) and rearranging them slightly, one obtains the following set of equations

$$1 + Ki\left[\frac{4}{N_{Re}N_{Pr}}\left(\frac{h}{r_o}\right) + \left(1-\frac{r_b}{r_o}\right)^2 - \frac{1}{4}\left(1-\frac{r_b}{r_o}\right)^4 - \frac{7}{24}\right.$$

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